A Simple Search Model with Employer Network

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Search and Matching Workshop in Asia-Pacific at National University of Singapore

Introduction

- Rogerson, Shimer, and Wright (2005) argue that even the earliest search models enhance our ability to organize observations on employment histories.
- Traditional literature often assumes that employers provide the same career prospects to workers, a presumption increasingly challenged by empirical evidence.
  - Cardoza et al. (2022) observed that approximately 20% of transition are between buyer and supplier in the Dominican Republic.
  - Komatsu (2023) documented that over 40% of job-to-job movements in Belgium occur within production networks.
- Through a simple search model, this paper studies how employer network structure can shape labor market outcomes by determining workers future prospect.

Introduction

- The paper marries the McCall search model with an employer network, where
  - employers only differ in network connections/positions;
  - the network connections between employers promise higher offer arrival rate
- Consequently, the network position of an employer confers a distinct expected value for future working opportunities – node value
- The node value is the key to understand the effect of employer network position on worker decisions and labor market outcomes.
  - Central employers have higher node values and are more attractive to workers
  - Node value is essentially an option asset and network centrality measure
  - Structure of the employer network matters

Related Literature

- Job search and social network
  - Montgomery (1991), Calvo-Armengol and Jackson (2004), Fontaine (2007), Bayer, Ross, and Topa (2008), Fontaine (2008), Cahuc and Fontaine (2009), Barwick et al. (2019)

  This paper focuses on the influence of connections and topology of employer network.

- Job search and employer network
  - Sorkin (2018), Cardoza et al. (2022), Komatsu (2023)

  I study the network properties of employer in terms of future working prospects.

Roadmap

Model

Equilibrium

Discussion of The Network Effect

Final Words

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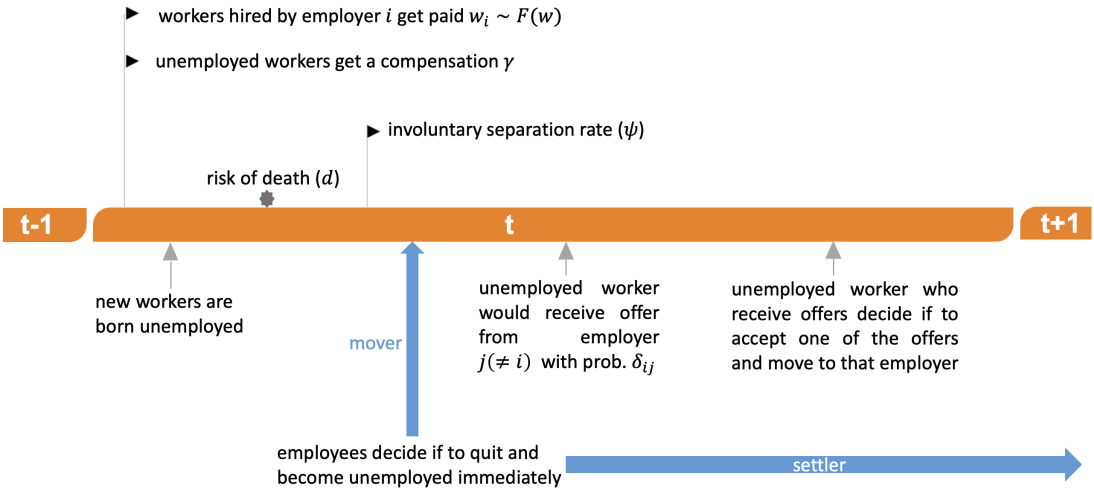
Model Setup

- Following McCall (1970), discrete time discounted by β
- Workers
  - homogeneous, risk-neutral, and exit labor market with prob. d
  - decisions made by maximizing lifetime income $\mathbb{E}_t \sum_{\tau=0}^{\infty} [\beta(1-d)]^{\tau} y_{t+\tau}$
  - the unemployed search randomly and may have multiple offers
- Employers
  - sent take-it-or-leave-it offer with a wage drawn independently from $F([0, B])$
  - finite number of employers only different in positions of the network $G(N_v, E)$
    - Set of nodes (employers): $N_v = \{1, 2, \ldots, i, \ldots, N\}$
    - Set of edges (connections): $E = \{e_{12}, \ldots, e_{2i}, \ldots, e_{iN}\}$.

Timeline


workers hired by employer i get paid $w_i \sim F(w)$
unemployed workers get a compensation γ
involuntary separation rate (ψ)
risk of death (d)
t-1
t
t+1
new workers are born unemployed
mover
unemployed worker would receive offer from employer $j(\neq i)$ with prob. δ_{ij}
unemployed worker who receive offers decide if to accept one of the offers and move to that employer
employees decide if to quit and become unemployed immediately
settler

Employer Network Setup

- Three assumptions that relate offer arrival rates to network connections

  1. the arrival rate δ_{ij} only depends on the connection between node i and node j, i.e.
     - for voluntarily unemployed workers (movers)

     $$\delta_{ij} = \begin{cases} \delta_L \in [0,1) & \text{if} \quad e_{ij} \notin E \\ \delta_H \in (\delta_L, 1] & \text{if} \quad e_{ij} \in E \end{cases}$$

     - for involuntarily unemployed workers

     $$\delta_{ij} = \begin{cases} \delta_L \in [0,1) & \text{if} \quad e_{ij} \notin E \\ m\delta_H + (1-m)\delta_L \in (\delta_L, 1] & \text{if} \quad e_{ij} \in E \end{cases}$$

     where m is network memory strength and $m \in [0,1]$.

  2. workers use their former employer's network connections in searching until they accept a new offer
  3. workers have no recall of employers they worked prior to their most recent one

Value Functions

- Employed workers at employer i:

$$V(i, w_i) = \max\{w_i + \beta(1-d)[(1-\psi)V(i, w_i) + \psi\Lambda_i], \Omega_i\}$$

- Voluntarily unemployed workers with $|C_i|$ offers in hand:

$$U^{\Omega}(i) = \max\{\gamma + \beta(1-d)\Omega_i, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots\}}_{|C_i| \text{ offers}},$$

- Involuntarily unemployed workers with $|C_i'|$ offers in hand:

$$U^{\Lambda}(i) = \max\{\gamma + \beta(1-d)\Lambda_i, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots\}}_{|C_i'| \text{ offers}},$$

- Inexperienced workers with $|C|$ offers in hand:

$$U^{0} = \max\{\gamma + \beta(1-d)\Lambda_0, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots\}}_{|C| \text{ offers}}.$$

Threshold Wage and Reservation Value

- A threshold wage in employer i

$$\eta_i = [1 - \beta(1-d)(1-\psi)]\Omega_i - \beta(1-d)\psi\Lambda_i,$$

  - $w_i \geqslant \eta_i \Rightarrow$ workers (settlers) choose to stay in the future
  - $w_i < \eta_i \Rightarrow$ workers (movers) voluntarily separate next period

- Unemployed workers compare the value of offers to their 'reservation value', $\bar{V}_i$, that depends on previous employer's network position, i.e.

$$\bar{V}_i = \begin{cases} \gamma + \beta(1-d)\Lambda_0, & \text{for inexperienced workers} \\ \gamma + \beta(1-d)\Omega_i, & \text{for voluntarily separated workers} \\ \gamma + \beta(1-d)\Lambda_i, , & \text{for involuntarily separated workers} \end{cases}$$

Premium from Network Position

- Unemployed worker separated from central employers have higher reservation value
- Offers from central employer are valued more
- An unemployed worker separated from employer i receives 2 offers each from employer j and employer j', she will work for j if

$$w_j + \beta(1-d)V(j, w_j) > w_{j'} + \beta(1-d)V(j', w_{j'}) > \bar{V}_i,$$

it is possible that

$$\beta(1-d)\Big[V(j, w_j) - V(j', w_{j'})\Big] > w_{j'} - w_j > 0$$

when employer j can provide an advantageous network position for future searching

Node Values

- Node values serve as a pivotal link between workers' decisions and the network structure of employers.
- They capture 3 key uncertainties faced by workers:
  - the wage associated with each offer
  - the number of offers may be received
  - the employers who send these offer

Node Values

- Strong node value for voluntarily separated workers:

$$\Omega_i = p_i(0)\left[\gamma + \beta(1-d)\Omega_i\right] + \sum_{n=1}^{N-1}\sum_{j=1}^{C_{N-1}^{n}} p_i(j \mid n)\max\Big\{\gamma + \beta(1-d)\Omega_i, \underbrace{\int_0^B \left[w_{i'} + \beta(1-d)V\left(i', w_{i'}\right)\right] dF\left(w_{i'}\right), \cdots}_{n \text{ offers arrived}}\Big\}$$

- Weak node value for involuntarily separated workers:

$$\Lambda_i = p_i'(0)\left[\gamma + \beta(1-d)\Lambda_i\right] + \sum_{n=1}^{N-1}\sum_{j=1}^{C_{N-1}^{n}} p_i'(j \mid n)\max\Big\{\gamma + \beta(1-d)\Lambda_i, \underbrace{\int_0^B \left[w_{i'} + \beta(1-d)V\left(i', w_{i'}\right)\right] dF\left(w_{i'}\right), \cdots}_{n \text{ offers arrived}}\Big\}$$

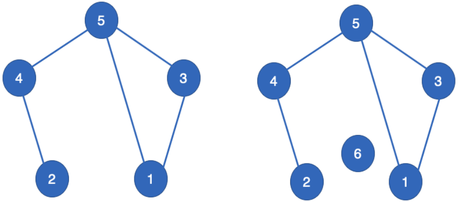
- Given the network structure, the node value for each employer uniquely exists.

Properties of Node Values

Three determinants of node value:

- Offer arrival rate: an increase in the offer arrival rate elevates the node values.
- Wage distribution $F(w)$
  - The node values increase with the first order stochastic dominance (FOSD) of $F(w)$ and its mean-preserving spreads in risk.
  - Network position of an employer is essentially an option asset for workers.
- Employer network position
  - The node value of an employer is an increasing and convex function of it neighboring node values.
  - Node value is essentially a centrality measure of employer's position in the network.

Examples: node values and network positions



Network		1	2	3	4	5	6
	Ω	891.602	883.869	891.602	889.267	891.369	
(a)	Λ	876.818	858.696	876.818	874.181	883.246	
	η	68.402	72.705	68.402	68.382	65.286	
	Ω	880.473	873.275	880.473	878.322	880.158	808.285
(b)	Λ	865.812	849.404	865.812	863.386	871.791	808.285
	η	67.577	71.368	67.577	67.557	64.626	55.772

Roadmap

Equilibrium

- In equilibrium, the inflow of employer i, $W^{in}(i)$, is proportional to the employment at employer i, $W(i)$, such that

$$W^{in}(i) = (1 - xs_i)W(i),$$

where

  - survival rate: $x = (1 - d)(1 - \psi)$
  - settlers ratio of employer i: $s_i = \Big(1 - F_i(\eta_i)\Big) \Big/ \Big(1 - xF_i(\eta_i)\Big)$
  - $F_i(z) = Pr(w < z|i)$ is the wage distribution of the unemployed workers who accept the offer from the employer i.

Labor Flows and Employment

- Labor inflow to employer i consists of inexperienced workers and movers from other employers.

$$W^{in}(i) = I \cdot P_{0i} + (1-d) \sum_{j \neq i} \left[(1-s_j) P_{ji}^{\Omega} + \psi s_j P_{ji}^{\Lambda} \right] W(j)$$

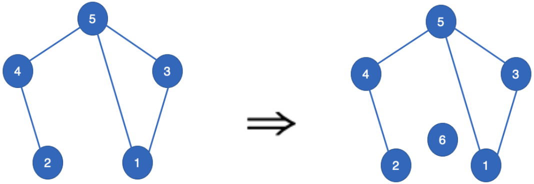
  - I: the total number of inexperienced workers including newly born workers and inexperienced workers from previous periods
  - P_{0i}: inflow probability of inexperienced workers
  - P_{ji}^{Ω}: inflow probability of workers voluntarily separated from employer j
  - P_{ji}^{Λ}: inflow probability of workers involuntarily separated from employer j

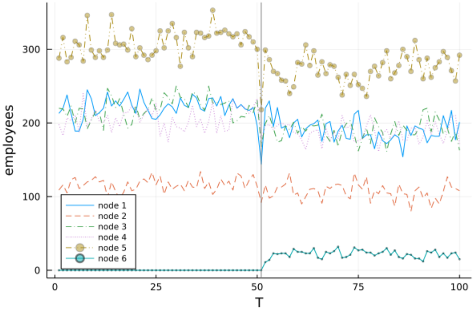
- The equilibrium employment for each employer $\hat{W} = [W(1), \ldots, W(N)]'$ satisfies

$$[1 - xs_1, 1 - xs_2, \cdots, 1 - xs_N] \hat{W} = H(\hat{W})$$

and $H(\hat{W})$ is the vector function representing the RHS of the flow function above.

Transitions after network structural shock





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Model

Equilibrium

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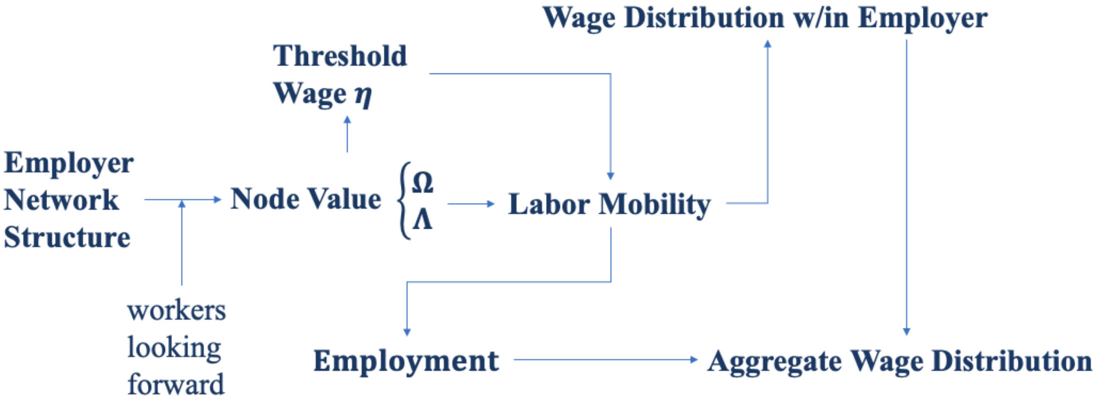
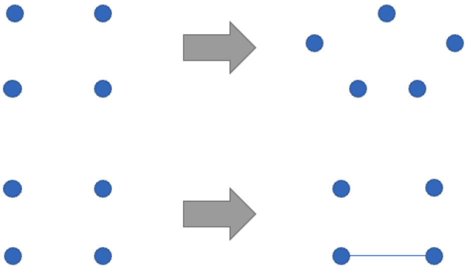
Final Words

How the network structure matters?



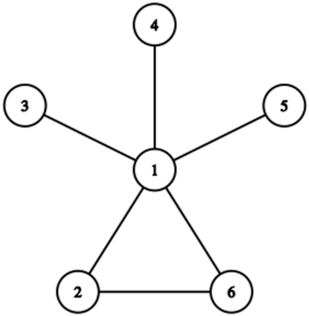
Network Structure and Node Values

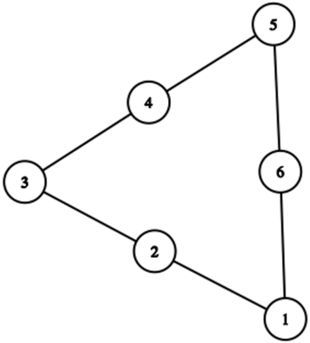


- The node values in a network weakly increase with additional edges.
- The effect of an additional node on node values is ambiguous depending on the original network structure.

Network Structure and Node Values

- Employer network structure affects labor market outcomes via node values.





Roadmap

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Main Takeaways

- Employer network structure matters to forward looking workers when it is related to future prospect of search and matching.
- Employer network structure influence the outcomes of labor market via node values that are essentially option assets and network centrality measures.

Insights

- Employer network position that promise better pecuniary opportunities poses a trade-off to workers in terms of current wage and future wages.
- Generally, wage cuts upon transitions may be caused by the pecuniary prospect resulted from certain employer characteristics.

Final Words

Further questions

- How do we know workers are moving for pecuniary reasons? Do they really matter?

  Need to quantitatively identify the motivations for transitions

- Do movers for pecuniary reasons have consistent wage dynamics after transitions?

  Need to track employment records and relate them to transition motivations

- How are employers distinct in providing better future to workers?

  Need to relate employer features to wages and future opportunities for workers

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My Job Market Paper answers these questions by

- Identifying the reasons for transitions using linked survey-administrative data of U.S.
- Relating transition motivations to earnings dynamics and subsequent transitions
- Emphasizing "steppingstones" employers as another pecuniary motivation for the future path of earnings

Appendix

Transition Probabilities

$$P_{0i} = \delta_L \Bigg\{ \sum_{n=0}^{N-1} \sum_{k=1}^{\binom{N-1}{n}} p_1(k \mid n) \int \cdots \int_{D_1} f(w_{i_1}, \cdots, w_{i_n}, w_i) dw_{i_1} \cdots dw_i \Bigg\},$$

$$P_{ji}^{\Omega} = \delta_{ji} \Bigg\{ \sum_{n=0}^{N-1} \sum_{k=1}^{\binom{N-1}{n}} p_2(k \mid n) \int \cdots \int_{D_2} f(w_{i_1}, \cdots, w_{i_n}, w_i) dw_{i_1} \cdots dw_i \Bigg\},$$

$$P_{ji}^{\Lambda} = \delta'_{ji} \Bigg\{ \sum_{n=0}^{N-1} \sum_{k=1}^{\binom{N-1}{n}} p_3(k \mid n) \int \cdots \int_{D_3} f(w_{i_1}, \cdots, w_{i_n}, w_i) dw_{i_1} \cdots dw_i \Bigg\},$$

where $p_z(k|n)$ ($z = 1, 2, 3$) is the probability of the kth combination conditional on n offers arrived besides the offer from employer i, $f(w_{i_1}, \cdots, w_{i_n}, w_i) = f(w_{i_1}) \cdots f(w_{i_n}) f(w_i)$ is the joint probability density of these $n + 1$ offers.

Transition Probabilities

D_z $(z = 1, 2, 3)$ is the domain where the unemployed worker to move to the employer i, i.e.

$$D_1 = \Big\{(w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_0 < w_i + \beta(1-d)V(i, w_i) \quad \&$$

$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n})\} < w_i + \beta(1-d)V(i, w_i)\Big\}$$

$$D_2 = \Big\{(w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_j^{\Omega} < w_i + \beta(1-d)V(i, w_i) \quad \&$$

$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n},)\} < w_i + \beta(1-d)V(i, w_i)$$

$$D_3 = \Big\{(w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_j^{\Lambda} < w_i + \beta(1-d)V(i, w_i) \quad \&$$

$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n})\} < w_i + \beta(1-d)V(i, w_i)\Big\}$$

where i_n indicates one of the n arrived offers dominated by the offer from employer i.