A Simple Search Model with Employer Network

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Abstract

This paper examines how employer network structures influence labor market outcomes using a simple search model. Employers are represented as network nodes, with edges reflecting higher job offer arrival rates. We establish the existence and uniqueness of a 'node value', the expected lifetime value of using an employer's network position for future job search. This node value has proved to be the key that connects workers decisions and employer network. Employer's node value has two features: (1) Akin to an option asset, it increases with the first-order stochastic dominance of wage distribution and mean-preserving risk spreads. (2) Similar to some network centrality measures, node value rises with the number of employer's connections and the node values of connected employers. As a result, employer network structures directly impact employment, labor mobility, and wage distributions, also explaining phenomena like worker mobility to lower-wage positions. This model underscores the importance of network topology in shaping labor market dynamics.

Keywords: Job search; Labor mobility; Employer network

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1 Introduction

Since Stigler (1961)'s foundational work, the search model has become a cornerstone in the economic analysis of wage dynamics, employment, and labor mobility.[1] Rogerson, Shimer, and Wright (2005) emphasize that search models significantly advance our understanding of the labor market. However, the traditional literature often assumes that employers are homogeneous to workers in providing career prospects, a presumption increasingly challenged by empirical evidence. Notably, Cardoza et al. (2022) observed that approximately 20% of transition are between buyer and supplier in the Dominican Republic. Komatsu (2023) documented that over 40% of job-to-job movements in Belgium occur within such production networks. This variation suggests that the network positions of employers and the network structure might play critical roles in shaping labor market dynamics. Therefore, this study seeks to unravel the impact of an employer's network position and the network topology on labor market outcomes.

This research marries the seminal search model by McCall (1970) with an employer network to investigate labor market dynamics. In particular, we conceptualize the employer network as a graph where nodes represent employers, interconnected by edges that signify potential employment links. This model envisages employers as islands, creating an environment where employed individuals receive wages and unemployed individuals await job offers from other islands. The offer arrival rate between two employers depends on the existence of connections between employers, with a higher offer arrival rate observed when a connection is present. Unlike the island model by Lucas Jr and Prescott (1974), our methodology abstracts from the specifics of employer production processes and the influence of worker externalities on wage determination. This strategic abstraction allows the study to zero in on the implications of an employer's network position and the broader network architecture.

Within this framework, the network position of an employer confers a distinct expected value for future employment opportunities associated with that employer, a concept we term as 'node value'. This node value emerges as the cornerstone of our model, encapsulating three key uncertainties faced by workers: the number of offers received, the employers behind the received offers, and the wage associated with each offer. Consequently, the structure, or topology, of the network significantly influences the dynamics of the labor market. It does so by affecting the node values assigned to each employer, which in turn impacts critical outcomes

[1] To name a few, sequential search by McCall (1970) and Mortensen (1970), random search by Pissarides (1985), and directed search by Moen (1997).

like wage distribution and overall employment levels.

In analyzing the influence of employer network, it is crucial to understand node values, as they not only reflect employers' positions within the network but also guide workers' forward-looking decisions. First, we demonstrate the existence of a unique node value for each employer within any given network structure. This result paves the way for further theoretical and numerical analyses. Second, we prove the parallels between the node values and the option assets. Specifically, we argue that either the first-order stochastic dominance of wage distribution, or its transformation into mean-preserving spreads, augments an employer's node value. Moreover, we posit that the node value resembles some network centrality measures and the intercentrality measure in a network game analyzed by Ballester, Calvó-Armengol, and Zenou (2006), highlighting the importance of an employer's number of connections and the node values of the connected employers as key determinants.[2] We prove that an employer's node value escalates with an increase in both the degree of connectivity and the node values of other employers, especially the connected ones.

Our model introduces a paradigm shift from conventional search models in two main aspects with node values. First, upon receiving job offers, workers evaluate these opportunities based on 'reservation values' as opposed to the 'reservation wage' in standard search models. This reservation value goes beyond mere wage considerations to include the position in the network of the prospective employer. Consequently, employers located at the central positions within the network command greater appeal due to their potential to offer superior future prospects, manifested as higher node values.

The model, therefore, seeks to explain the mobility of workers towards jobs with wages that are below their stated reservation wage or their wages from previous employers[3]. Contrary to previous interpretations that attribute this mobility to non-wage compensation provided by employers (Hall and Mueller (2018), Sorkin (2018)), this paper proposes an alternative perspective. It posits that the evaluation of wages, when considered alongside the trade-off between spot wage and anticipated future wages, remains a pertinent framework for understanding these employment transitions. In this context, the sequential auction model of Postel-Vinay

[2]In network science, 'degree' refers to the number of connection of one node in an undirected network, and 'indegree(outdegree)' in directed network. A 'neighbor indicates a connected node.

[3]Hall and Mueller (2018) find that unemployed workers sometimes accept offers with wages below their previously stated reservation wages. Sorkin (2018) documents that declining earnings upon transitions are an important feature in the US labor market.

and Robin (2002), which links mobility involving wage reduction to prospective wage growth within the employer, serves as a reference point. However, our model emphasizes the role of employers' network connections in offering future income prospects.

The second divergence of our model from existing labor market search models lies in its treatment of employer network structures that directly determine labor market outcomes, including observed wage distribution, labor mobility, and employment. Specifically, our model posits that employers who occupy central positions within network structures facilitate greater labor mobility.

Furthermore, our analysis extends to the impact of structural changes within the employer network - such as variations in the number of nodes (employers), edges (connections between employers), and the overall network topology - on labor market outcomes. The model shows how shocks to the network's structure, termed network structural shocks, affect these outcomes by altering the node values assigned to each node (employer) within the network. Specifically, adding a new edge to a network would weakly increase all node values in the network, while the effect of an additional node on node values depends on the specific topology of the network. This complexity is further magnified when considering wage distributions, as the network's restructuring influences labor flows among employers in an endogenous manner.

Literature Review: This paper revisits a large literature that incorporates network factors with job search. After the seminal work of Montgomery (1991) and Calvo-Armengol and Jackson (2004), many researchers have delved into the relation between job search and the network on the worker side, the social network. Studies such as those by Fontaine (2007), Bayer, Ross, and Topa (2008), Fontaine (2008), Cahuc and Fontaine (2009) and Barwick et al. (2019), underscore the importance of social networks in the labor market.[4] Social networks are empirically documented to facilitate search and matching for workers through referrals (Dustmann et al., 2016; Lester, Rivers, and Topa, 2021) and information passing (Glitz, 2017; Arbex, O'Dea, and Wiczer, 2019; Caldwell and Harmon, 2019; Carrillo-Tudela, Kaas, and Lochner, 2023). Although this paper acknowledges the significant effect of social networks on the labor market, we diverge by focusing on the connections and topology of the employer network. Unlike the contagious effect in (Calvo-Armengol and Jackson, 2004), the workers are homogeneous and their behaviors in our model do not affect other workers through the employer network.

In addition to our study, the recent literature, including Cardoza et al. (2022) and Komatsu

[4] Jackson (2006) provides a complete survey of the theory on the economics of networks at that time.

(2023), highlights the impact of employer networks on labor market dynamics. Haltiwanger et al. (2018) documented that worker transitions between employers significantly contribute to overall labor mobility in frictional labor markets. Sorkin (2018). Unlike previous studies that focus on the input-output relationships or firm-to-firm transactions, our approach abstracts the employer network by considering heterogeneous offer arrival rates determined by an employer's network position. We contribute to the existing theory by examining how the structure of the employer network influences worker decisions and eventual outcomes in labor market.

The remainder of this paper is organized as follows. Section 2 introduces the model and elucidates the key elements that link the network structure with the decision-making processes of the workers. Section 3 employs numerical analyses to investigate the steady-states of the model and assesses the impact of structural changes within the network on this equilibrium. Section 4 explores how variations in network structure influence wage distribution and labor mobility. Section 5 concludes.

2 Model

2.1 Environment

Agents: The economy has N employers and a pool of homogeneous workers. We assume that the entry rate equals the exit rate (mortality) of the labor force, so the pool of workers is stable. To isolate the network structure's effects, this model abstracts away from the diverse characteristics of employers, focusing solely on their positions within the network to study the network's impact on employment dynamics. That is, employers are differentiated exclusively by their network positions.

Following McCall (1970), we assume workers are rational and risk-neutral, aiming to maximize the expected present value of their lifetime income, represented as:

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} [\beta(1-d)]^{\tau} y_{t+\tau},$$

where β denotes the discounting rate, d is the labor force exit rate (or mortality risk), y_t is the income at time t. The model posits that income is contingent upon employment status - a constant γ for the unemployed, and a variable wage w_i for those employed by employer i, which is independently drawn from a non-degenerate distribution $F(x)$ and $w \in [0,B]$.[5] The

[5] $F(x)$ satisfies $F(x) = Prob(w \leq x)$, with $F(\underline{w}) = 0$, $F(B) = 1$, and its pdf. $f(w) > 0$ for $\forall w \in [\underline{w}, B]$.

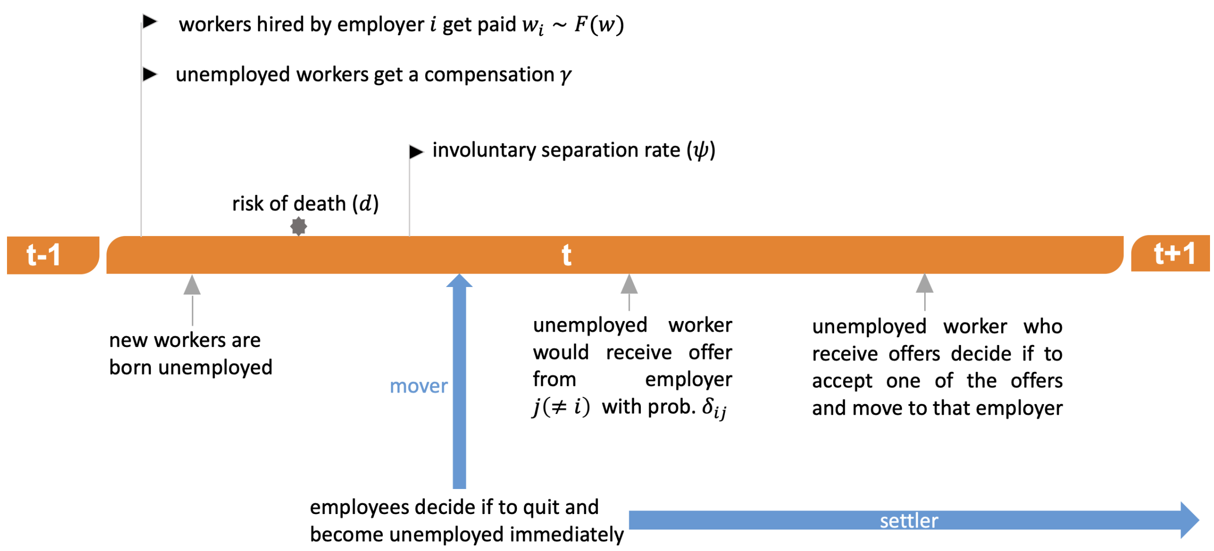


Figure 1: Decision timeline

wage of the employer i remains unchanged after the offer is accepted.

Timing of Actions: As shown in Figure 1, at the beginning of each period, workers are compensated with w or γ, depending on their employment state. New workers entering the job market are unemployed. However, a (n) decreasing(increasing) population would not change the results of this paper. In addition to a mortality risk, every worker faces an exogenous separation risk ψ. If alive and not separated, a worker in employer i decides whether to quit and enter the job market immediately. We will explain that the worker is a 'mover' if she voluntarily quits, and a 'settler' otherwise.[6] Then unemployed workers in the job market may receive offers from other employers except her previous one.[7] After accepting one of the arrived offer(s), she will move to the employer with no commuting cost and start working next period.

Employer Network: Employer network is characterized by a graph $G(N_v, E)$, which is captured by an adjacency matrix $A = [a_{ij}]_{N\times N}$, where $a_{ij} = 1$ if there is a direct link between employers

[6]This model abstracts from on-the-job search that is prevalent in many search models (e.g. Pissarides (1994), Burdett and Mortensen (1998)) for two reasons. First, we aim to study the role of the employer network position in workers' decisions. Adding on-the-job search for workers would have marginal benefit for this purpose but mix the workers who flow in for network connections and for higher wages. Second, the framework still accommodates job-to-job transitions observable within the discrete-time setup, facilitating an examination of how network structures influence worker mobility.

[7]The assumption of recalling from the previous employer would not affect the main results.

i and j, and 0 otherwise.[8]

The model introduces three foundational assumptions that facilitate an understanding of how the network structure influences the labor market, emphasizing the role of network positions in shaping job offer probabilities and employment decisions.

1. Offer Arrival Rate: Workers separated from the employer i can receive an offer from employer $j(\neq i)$, with the probability δ_{ij} influenced by the network connectivity between the two employers. This probability varies, indicating that a worker's network position significantly impacts their job prospects. For voluntarily separated workers (movers),

$$\delta_{ij} = \begin{cases} \delta_L \in [0,1) & \text{if} \quad a_{ij} = 0 \\ \delta_H \in (\delta_L, 1] & \text{if} \quad a_{ij} = 1 \end{cases},$$

For workers involuntarily separated from their previous employers,

$$\delta_{ij} = \begin{cases} \delta_L \in [0,1) & \text{if} \quad a_{ij} = 0 \\ \delta_m = m\delta_H + (1-m)\delta_L \in (\delta_L, 1] & \text{if} \quad a_{ij} = 1 \end{cases},$$

where $m \in [0,1]$ is network memory strength.

2. Network Utilization in Job Search: Workers can leverage their former employer's network connections for job search until they accept a new offer, underscoring the lasting impact of previous employment on current job search efforts.

3. Limited Network Memory: Workers do not recall employers beyond their most recent one and thus cannot leverage previous employers' network connections.

Remarks: (1) The first assumption illustrates the mapping of the employer network graph to the heterogeneous arrival rate of the offers. If involuntarily separated, the network connections are weaker unless $m = 1$. If $m = 0$, employed workers have no incentive to quit their jobs. (2) The second assumption implies that a worker may receive an offer from the same employer that she previously rejected. This is because we make employers only differ in their network positions. In practice, workers are likely to receive an offer from another employer that shares similar features with the one they previously rejected. This assumption streamlines the stationary value

[8] $N_v = \{1, \ldots, i, \ldots, N\}$ is the set of nodes (employers), and E is the set of edges between two nodes. This paper focuses on the unweighted and undirected network structure, but we can easily extend it to the weighted directed network without hurting the main results of the paper.

about network positions that we will discuss later. (3) The Markov-like assumption simplifies the computation and can be extended with a state of history.

Workers States and Transitions: As in Figure 2, each period begins with workers positioned at various nodes (representing employers), where they may assume one of three distinct statuses: 'settler', 'mover', or 'unemployed'. In particular, new workers ('inexperienced') in the labor market lack prior employment experience and, as such, are assumed to search from a hypothetical employer node 'zero', devoid of any network connections. Consequently, given N employers within the economic framework, the model identifies a total of $3N+1$ unique worker states per period. The decision to remain within an employer or to resign is predicated on a comparison between the current wage and the expected value attributed to the employer's network position. A settler remains indefinitely with a single employer, whereas a mover opts to resign after a solitary period of employment in pursuit of new opportunities, leveraging the network connections of the previous employer.[9] Transitions between states are visually represented in the figure: solid arrows signify immediate subsequent state transitions, while dashed arrows indicate potential transitions. In contrast, newly born workers are free to transition to any employer node, as highlighted by the red dashed arrows.

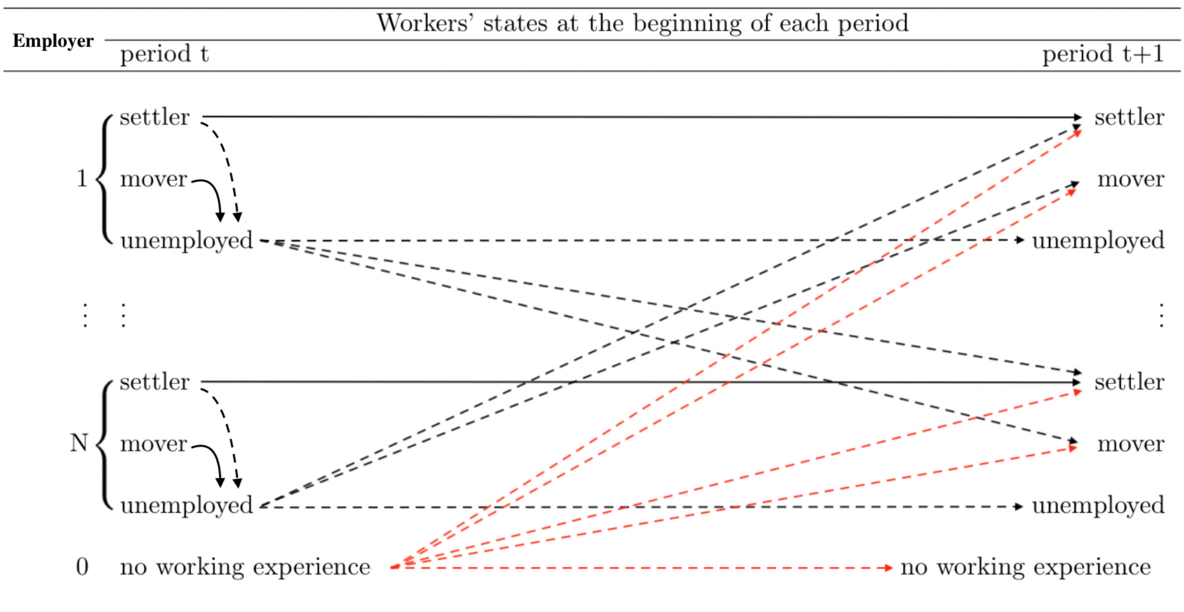


Figure 2: Workers' states and flows

[9] As will be explained later, given the employment status, the policy rule is time-invariant because the network structure and wage distribution are fixed.

2.2 Value Functions

The search has stationary values over time, capturing the interaction between wages, employment status, and network positions. The value function for employed workers at employer i each period is:

$$V(i, w_i) = \max\{w_i + \beta(1-d)[(1-\psi)V(i, w_i) + \psi\Lambda_i], \Omega_i\}, \tag{1}$$

The first term on the right is the value if the worker stays, capturing the expected value of involuntary separation, Λ_i. The second term, Ω_i, is the expected value if she voluntarily quits. These expected values, tied to the employer's network position, are thus termed the 'strong node value' (Ω_i) and 'weak node value' (Λ_i).

The values for unemployed workers rely on their particular network connections. Denote $U^{\Omega}(i)$ ($U^{\Lambda}(i)$) as the value for voluntarily (involuntarily) separated workers, then

$$U^{\Omega}(i) = \max\{\gamma + \beta(1-d)\Omega_i, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots}_{|\mathscr{C}_i| \text{ offers}}\}, \tag{2}$$

$$U^{\Lambda}(i) = \max\{\gamma + \beta(1-d)\Lambda_i, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots}_{|\mathscr{C}_i'| \text{ offers}}\}, \tag{3}$$

where $\mathscr{C}_i$ ($\mathscr{C}_i'$) is the sets of arrived job offers for the voluntarily (involuntarily) unemployed. Should no offers be received, or all received offers are declined, the resulting values default to $\gamma + \beta(1-d)\Omega_i$ for voluntarily unemployed workers and to $\gamma + \beta(1-d)\Lambda_i$ for involuntarily unemployed workers.

Let U^0 be the value for inexperienced workers, including new workers and workers who have never worked after the entry.

$$U^0 = \max\{\gamma + \beta(1-d)\Lambda_0, \underbrace{w_{i'} + \beta(1-d)V(i', w_{i'}), \cdots}_{|\mathscr{C}| \text{ offers}}\}. \tag{4}$$

where $\mathscr{C}$ is the set of offers arrived. Equation 4 caters to newly entered workers in the job market, attributing their expected unemployed value to $\gamma + \beta(1-d)\Lambda_0$, as they have no network connection from employers.

2.3 Threshold Wage and Reservation Value

A notable implication from the value functions above is the introduction of a 'threshold wage', η_i, for employer i, beyond which workers opt to stay (settlers) or leave (movers). This threshold wage, defined as:

$$\eta_i = [1-\beta(1-d)(1-\psi)]\Omega_i - \beta(1-d)\psi\Lambda_i, \tag{5}$$

is demonstrably positive, ensuring a decision-making framework that deviates from conventional models by considering both the wage and the employer's network position in the job acceptance process.[10]

Compared to the conventional McCall model, this model transitions from focusing on a 'reservation wage' to a 'reservation value', $\bar{V}_i$. This reservation value for unemployed workers, separated from employer i, is delineated as:

$$\bar{V}_i = \begin{cases} \gamma + \beta(1-d)\Lambda_0, \text{ for inexperienced workers} \\ \gamma + \beta(1-d)\Omega_i, \text{ for voluntarily separated workers} \\ \gamma + \beta(1-d)\Lambda_i,, \text{ for involuntarily separated workers} \end{cases}. \tag{6}$$

Importantly, the reservation value allows for a scenario where workers may accept offers with wages lower than their reservation wages, or switch employers with declining wages, should the network position of the prospective employer justify such a decision. For instance, consider an unemployed worker separated from employer i who received two offers each from employer j and employer j'. The worker will be working next period if and only if the following holds:

$$\max\left\{\left(w_j + \beta(1-d)V(j,w_j) - \bar{V}_i\right), \left(w_{j'} + \beta(1-d)V(j',w_{j'}) - \bar{V}_i\right)\right\} > 0,$$

and reservation wage plays no role in worker's decision.

Furthermore, a worker can reject an offer from employer j' and go to employer j if

$$w_j + \beta(1-d)V(j,w_j) > w_{j'} + \beta(1-d)V(j',w_{j'}).$$

In this case, even if $w_{j'} > w_j$, but $w_{j'} - w_j < \beta(1-d)\left[V(j,w_j) - V(j',w_{j'})\right]$, the superficially better offer from j' might not be preferable as employer j can provide a significantly advantageous network position for future searching. The network position can make the expected value

[10]Since

$$f(\psi) \in [0, \beta(1-d)], f'(\psi) > 0, f'' < 0$$

and $\Omega_i \geq \Lambda_i$ by definition,

$$\Omega_i > \frac{\beta(1-d)\psi}{1-\beta(1-d)(1-\psi)}\Lambda_i = f(\psi)\Lambda_i,$$

so $\eta_i > 0$.

sufficiently high to surpass the immediate wage difference, leading to the worker's decision to decline the offer from j' in favor of transitioning from employer i to j. These conditions articulate the necessary and sufficient criteria that explain scenarios in which workers may favor offers with lower wages or switch employers despite a wage reduction.

2.4 Node Values and Their Properties

Node values serve as a pivotal link between workers' decisions and the network structure of employers. Specifically, they encapsulate three key uncertainties faced by workers: the number of offers received, the employers behind the received offers, and the wage associated with each offer. These uncertainties are reflected by both strong and weak node values as below

$$\begin{aligned}\Omega_i = & \, p_i(0)\left[\gamma+\beta(1-d)\Omega_i\right]+ \\ & \sum_{n=1}^{N-1}\sum_{k=1}^{\binom{N-1}{n}} p_i(k\mid n)\max\{\gamma+\beta(1-d)\Omega_i, \underbrace{\int_0^B\left[w_{i'}+\beta(1-d)V\left(i',w_{i'}\right)\right]dF\left(w_{i'}\right),\cdots}_{n \text{ offers arrived}}\},\end{aligned} \tag{7}$$

$$\begin{aligned}\Lambda_i = & \, p'_i(0)\left[\gamma+\beta(1-d)\Lambda_i\right]+ \\ & \sum_{n=1}^{N-1}\sum_{k=1}^{\binom{N-1}{n}} p'_i(k\mid n)\max\{\gamma+\beta(1-d)\Lambda_i, \underbrace{\int_0^B\left[w_{i'}+\beta(1-d)V\left(i',w_{i'}\right)\right]dF\left(w_{i'}\right),\cdots}_{n \text{ offers arrived}}\},\end{aligned} \tag{8}$$

Here $\int_0^B\left[w_{i'}+\beta(1-d)V\left(i',w_{i'}\right)\right]dF\left(w_{i'}\right)$ represents the expected value conditional on receiving the offer from employer i.[11]. The probability $p_i(0)$ and $p_i(k|n)$ correspond to the case of no offer and n offers arrived. The latter specifies a set of employers from which the worker has received offers, denoted as $\mathscr{C}_i$. Then, for n received offers, the total possible combinations of these offers are given by $\binom{N-1}{n}=\frac{(N-1)!}{(N-1-n)!n!}$, since the maximum number of offers that can arrive is $N-1$. k is the index that denotes one of the combinations. The conditional probabilities are

$$p_i(0)=\prod_{j\neq i}^{j\in N_v}\{1-\delta_{ij}\}=\prod_{j\neq i}^{j\in N_v}\{1-[a_{ij}\delta_H+(1-a_{ij})\delta_L]\}, \tag{9}$$

$$p_i(k|n)=\left(\prod_{z\in\mathscr{C}_i}\delta_{iz}\right)\cdot\left(\prod_{z'\in N_v\setminus(\mathscr{C}_i\cup i)}(1-\delta_{iz'})\right), \tag{10}$$

[11]Without loosing generality, we let $\underline{w}=0$.

To explore the properties of node values, we introduce a $2N \times 1$ vector with node values as elements, denoted $\Xi = [\Omega_1, ..., \Omega_N, \Lambda_1, ..., \Lambda_N]'$, and a nonlinear function $G(\cdot)$ that satisfies the right sides of equations 7 and 8. Our investigation is centered around the existence of a fixed point Ξ for the function $G(\cdot)$ such that:

$$\Xi = G(\Xi). \tag{11}$$

Proposition 1 below establishes the existence of a fixed point for 11, ensuring that within any directed or undirected network structure, each employer corresponds to a unique pair of node values denoted by (Ω, Λ).

Proposition 1. *The node values for each employer exist uniquely within any network structure.*

Proof. The idea of proving is to construct a Jacobian matrix of $G(\cdot)$. The probability coefficients and partial derivatives (g_1' and g_2') will be shown to be strictly less than one. So, the Jacobian matrix is strictly less than one under some natural matrix norm. The contraction mapping will guarantee the unique existence of the fixed point in equation 11.

There are 2^{N-1} maximizing functions on the right side of equations 7 and 8. Take one of the combinations as an example: $\max(\vec{v}) = \max\{v_0, v_1, ..., v_n\}$ denotes the case where n offers arrive and the worker has to choose the best one. $v_0 = \bar{V}_i$ is the reservation value of the worker from employer i, $v_j (j \neq i)$ is the value of offers from other employers such that $v_j = \frac{1+\beta(1-d)\psi}{1-\beta(1-d)(1-\psi)} \mathbb{E}(w) + g(\Omega_j, \Lambda_j)$, where $g(\Omega_j, \Lambda_j)$ is an increasing and convex function defined in the proof of Proposition 3.

Define a 'softmax' function: $\mu_\alpha(\vec{v}) = (v_0^\alpha + v_1^\alpha + \cdots + v_n^\alpha)^{1/\alpha}$. By Hölder's inequality,

$$\max(\vec{v}) \leq (v_0^\alpha + v_1^\alpha + \cdots + v_n^\alpha)^{1/\alpha} \leq (n+1)^{1/\alpha} \max(\vec{v}) \leq N^{1/\alpha} \max(\vec{v}) \tag{12}$$

hence, the softmax function $\mu_\alpha(\vec{v})$ converges to $\max(\vec{v})$ when $\alpha \to \infty$.

Construct $J_G(x)$ as the Jacobian matrix of the first partial derivatives of the function $G(x)$, based on the softmax function with large α. Using the equation (20) and (22) in the proof of Proposition 3, we can derive the diagonal elements of row i in $J_G(\hat{V})$, which is the first partial derivative of V_i:

$$\begin{aligned} &\beta(1-d)p_i(0) + \beta(1-d) \sum_{n=1}^{N-1} \sum_{j=1}^{C_{N-1}^n} p_s(j|n) \Big(\frac{v_0^\alpha}{v_0^\alpha + v_1^\alpha + \cdots + v_n^\alpha}\Big)^{\frac{\alpha-1}{\alpha}} \\ &< \beta(1-d)\Big[p_i(0) + \sum_{n=1}^{N-1} \sum_{j=1}^{C_{N-1}^n} p_i(j|n)\Big] = \beta(1-d) < 1 \end{aligned} \tag{13}$$

and the element of column j ($j \leq N$) and row i is:

$$\begin{aligned}
&\beta(1-d)g_1'(\Omega_j,\Lambda_j)\sum_{n=1}^{N-1}\sum_{j=1}^{C_{N-1}^n} p_i(j|n)\big(\frac{v_j^\alpha}{v_0^\alpha+v_1^\alpha+\cdots+v_n^\alpha}\big)^{\frac{\alpha-1}{\alpha}} \\
&<\beta(1-d)\sum_{n=1}^{N-1}\sum_{j=1}^{C_{N-1}^n} p_i(j|n)<1,
\end{aligned} \tag{14}$$

The first inequality uses $g_1'(\Omega_j,\Lambda_j)<1$ as shown in the proof of Proposition 3. The results are the same if $N<j\leq 2N$.

Hence, all elements of $J_G(\hat{V})$ are strictly less than 1. We can find a small positive real number ε and a constant $\rho=1-\varepsilon$ such that

$$\parallel J_G(\hat{V}) \parallel_\infty < \rho < 1 \tag{15}$$

where we choose the infinite matrix norm (L_∞ norm). Therefore, $G(\hat{V})$ is a contraction mapping and has a unique fixed point $\hat{V}^*$. We can compute $G(\hat{V}^*)$ by iterating the G function because

$$|\hat{V}^{(t+2)}-\hat{V}^{(t+1)}| = |G(\hat{V}^{(t+1)})-G(\hat{V}^{(t)})| \approx |J_G(\hat{V})(\hat{V}^{(t+1)}-\hat{V}^{(t)})| \leq \rho|\hat{V}^{(t+1)}-\hat{V}^{(t)}|$$

■

Next, we discuss the effect of 3 determinants of node value: offer arrival rate, wage distribution, and employer network position. First, an increase in the offer arrival rate directly elevates the node values, as it reflects an elevated likelihood of favorable outcomes. This assertion comes from the construction of the node values in equations 7 and 8. To illustrate, consider the distinction between Ω_i and Λ_i, which is based solely on the conditional probabilities p_i and p_i'. The relation $p_i(\cdot|n) \geq p_i'(\cdot|n)$ suggests that $\Omega_i \geq \Lambda_i$, with equality manifesting exclusively when $m=1$.

Second, the influence of wage distribution $F(w)$ on the node values follows Proposition 2 below.

Proposition 2. *The first-order stochastic dominance (FOSD) of $F(w)$ increases the node values. Mean-preserving spread in risk of $F(w)$ increases node values.*

Proof. The proof makes use of the weak monotonicity of $V(i,w_i)$ in w_i. If F_1 first-order stochastically dominates F_2, $F_1(x)<F_2(x), \forall x$. And since $V(s',w_{s'})$ is weakly increasing convex function in terms of $w_{s'}$,

$$\begin{aligned}
&\int_0^B [w_{s'}+\beta(1-d)V(s',w_{s'})]dF_1(w_{s'}) = \mathbb{E}(w)+\beta(1-d)\int_0^B V(s',w_{s'})dF_1(w_{s'}) \\
&> \mathbb{E}(w)+\beta(1-d)\int_0^B V(s',w_{s'})dF_2(w_{s'}) = \int_0^B [w_{s'}+\beta(1-d)V(s',w_{s'})]dF_2(w_{s'})
\end{aligned}$$

Let F_2 be the mean-preserving spread of F_1. Then

$$x \sim F_1, \quad y \sim F_2, \quad y = x + \varepsilon, \quad \mathrm{E}(\varepsilon|x) = 0$$

By Jensen's inequality,

$$\int_0^B [w_{s'} + \beta(1-d)V(s', w_{s'})]dF_1(w_{s'}) = \mathbb{E}(w) + \beta(1-d)\int_0^B V(s', w_{s'})dF_1(w_{s'})$$
$$< \mathbb{E}(w) + \beta(1-d)\int_0^B V(s', w_{s'})dF_2(w_{s'}) = \int_0^B [w_{s'} + \beta(1-d)V(s', w_{s'})]dF_2(w_{s'})$$

■

Proposition 2 highlights that the network position of an employer is essentially an option asset for workers. FOSD implies that an option offering universally higher (or at least not lower) payoffs in such a risk-neutral world would indeed be valued more highly. Pricing theory shows that the value of an option is an increasing function of the variance in the price of the underlying asset. Proposition 2 is intuitive considering that workers have the prerogative to accept offers only from the right tail of the distribution. Under a mean-preserving increase in risk, the higher incidence of better wage offers increases the value of holding out for more opportunities through their network connections in subsequent periods, while the higher incidence of very bad offers is not detrimental as the option will not be exercised anyway.[12]

Third, the network position of an employer determines its node value, drawing parallels to the principle of centrality measures within network science. Centrality measures quantify the relative importance of a node within a network through two primary dimensions: (i) nodes with a higher degree of connections exhibit greater centrality; (ii) a node's centrality is further amplified by the high centrality of its connected nodes. In a similar vein, an employer's node value is augmented by an increase in connections, underscoring the pivotal role of network ties in enhancing expected value. Moreover, node values of other employers in the network influence each other, as elucidated in Proposition 3.

Proposition 3. *The node value of an employer is an increasing and convex function of its neighboring node values.*

Proof. The effect of neighboring nodes are captured in the term $\int_0^B [w_j + \beta(1-d)V(j, w_j)]dF(w_j)$, which can be written into $\frac{1+\beta(1-d)\psi}{1-\beta(1-d)(1-\psi)}\mathbb{E}(w) + g(\Omega_j, \Lambda_j)$, as shown below:

$$\int_0^B [w_{s'} + \beta(1-d)V(s', w_{s'})]dF(w_{s'}) = \int_0^B w_{s'}dF(w_{s'}) + \beta(1-d)\int_0^B V(s', w_{s'})dF(w_{s'}) \quad (16)$$

[12] Ljungqvist and Sargent (2018) addresses this points when introducing McCall's model in page 166.

$$\int_0^B V(s', w_{s'}) dF(w_{s'}) = \int_0^{\eta_{s'}} \Omega_{s'} dF(w_{s'}) + \int_{\eta_{s'}}^B \frac{w_{s'} + \beta(1-d)\psi\Lambda_{s'}}{1-\beta(1-d)(1-\psi)} dF(w_{s'})$$
$$= \Omega_{s'} F(\eta_{s'}) + \frac{\beta(1-d)\psi\Lambda_{s'}}{1-\beta(1-d)(1-\psi)}[1-F(\eta_{s'})] + \int_{\eta_{s'}}^B \frac{w_{s'}}{1-\beta(1-d)(1-\psi)} dF(w_{s'}) \tag{17}$$

Let $\Phi(\Omega_{s'}, \Lambda_{s'}) = \Omega_{s'} F(\eta_{s'}) + \frac{\beta(1-d)\psi\Lambda_{s'}}{1-\beta(1-d)(1-\psi)}[1-F(\eta_{s'})]$,

Replace eq.(17) into eq.(16),

$$\begin{aligned} &\int_0^B [w_{s'} + \beta(1-d)V(s', w_{s'})] dF(w_{s'}) \\ &= \int_0^B w_{s'} dF(w_{s'}) + \frac{\beta(1-d)}{1-\beta(1-d)(1-\psi)} \int_{\eta_{s'}}^B w_{s'} dF(w_{s'}) + \beta(1-d)\Phi(\Omega_{s'}, \Lambda_{s'}) \\ &= \frac{1+\beta(1-d)\psi}{1-\beta(1-d)(1-\psi)} \mathbb{E}(w) + g(\Omega_{s'}, \Lambda_{s'}) \end{aligned} \tag{18}$$

where

$$g(\Omega, \Lambda) = \beta(1-d)\Phi(\Omega, \Lambda) - \frac{\beta(1-d)}{1-\beta(1-d)(1-\psi)} \int_0^\eta w dF(w) \tag{19}$$
$$\eta = [1-\beta(1-d)(1-\psi)]\Omega - \beta(1-d)\psi\Lambda$$

and $\mathrm{E}(w) = \int_0^B w_{s'} dF(w_{s'})$,

$$0 < \frac{\partial g(\Omega, \Lambda)}{\partial \Omega} = \beta(1-d)F(\eta) < 1 \tag{20}$$

$$\frac{\partial^2 g(\Omega, \Lambda)}{\partial \Omega^2} = \beta(1-d)[1-\beta(1-d)(1-\psi)]f(\eta) > 0 \tag{21}$$

$$0 < \frac{\partial g(\Omega, \Lambda)}{\partial \Lambda} = \frac{\beta^2(1-d)^2\psi[1-F(\eta)])}{1-\beta(1-d)(1-\psi)} < 1 \tag{22}$$

$$\frac{\partial^2 g(\Omega, \Lambda)}{\partial \Lambda^2} = \frac{\beta^3(1-d)^3\psi^2 f(\eta))}{1-\beta(1-d)(1-\psi)} > 0 \tag{23}$$

$$\frac{\partial^2 g(\Omega, \Lambda)}{\partial \Lambda \partial \Omega} = -\beta^2(1-d)^2\psi f(\eta) < 0 \tag{24}$$

Hence, the function $g(\Omega, \Lambda)$ is increasing and convex with respective to Ω and Λ. ■

Proposition 3 illustrates that, as demonstrated numerically in the following subsection, a node connecting to fewer nodes with high node values may exhibit a greater node value compared to a node connecting to many low node values. This characteristic is pivotal for understanding the profound influences exerted by the positions and structures of the network. Importantly, this property makes node value comparable to some network centrality measures,

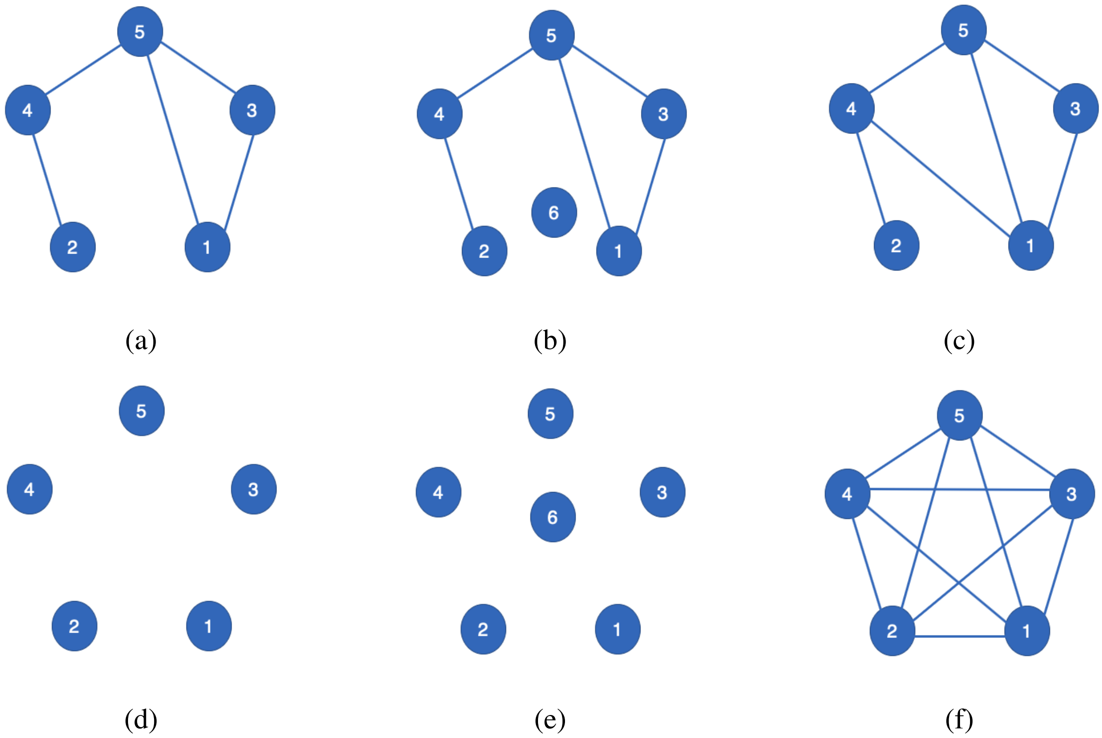


Figure 3: Network examples

such as eigenvector centrality and PageRank. Nevertheless, the node values within this search framework are influenced by the values of the neighboring nodes in a non-linear manner. [13]

2.5 Numerical Examples

This section presents examples to elucidate the properties discussed previously, followed by a career trajectory for a typical worker within an employer network structure, and a discussion on computational challenges encountered as the network scales.

2.5.1 Examples

To demonstrate the implications of Proposition 3, we reference the employer network structures depicted in Figure 3, analyzing their influence on node values and threshold wages.

Specific parameters are used for illustration: the wage distribution follows a uniform pattern, $w \sim U(0,B)$ with $B = 100$; the discount factor multiplied by the mortality risk is calculated as $\beta(1-d) = 0.98 \cdot 0.95 = 0.931$; the involuntary separation rate stands at $\psi = 0.5$; arrival rates

[13] The concept of nonlinear centrality has previously been explored within the realm of network science, as evidenced by research conducted by Tudisco and Higham (2019) and Arrigo and Tudisco (2019).

are differentiated as $\delta_H = 0.9$, $\delta_L = 0.1$; network memory strength is $m = 0.5$; and the unemployment compensation $\gamma = 0.15E(w) = 7.5$. The labor force is stable with the same mortality and entry rates. The networks are undirected.

Table 1 shows how the positions of the employer network are directly correlated with the node values. For example, node 2, being the most isolated, registers the lowest values for Λ and Ω. Nodes 1 and 3, due to their symmetrical positioning, share identical node values. Despite node 4's connectivity to two neighbors (nodes 2 and 5), its node value is adversely impacted by the low value of node 2, underscoring the principle that an isolated neighbor can diminish a node's value. In contrast, regular network structures (as depicted in graphs (d)-(f)) exhibit uniform node values because of their symmetrical configurations.

Although the concept of weak node values effectively ranks nodes according to the relative significance of their network positions, the interpretation of strong node values is not as straightforward. For instance, in graph (a) of Figure 3, node 5's network central position grants it the highest Λ, yet its Ω does not lead. This anomaly is attributed to the influence of neighboring nodes on the valuation function $g(\Omega,\Lambda)$, whose second mixed partial derivative with respect to Ω and Λ is negative.

Moreover, central nodes typically exhibit lower threshold wages, suggesting a propensity for workers to gravitate toward central employers. However, the hierarchy of threshold wages does not consistently mirror the ranking of node values. For instance, in graph (a), the most isolated node (node 2) commands the highest threshold wage. In contrast, the threshold wage for node 4, despite being the second most isolated, is lower than those associated with nodes 1 and 3. This divergence underscores the fact that threshold wages are influenced by a constellation of factors, including the discounting rate, the separation rate, and the strong and weak node values. Furthermore, network memory strength, denoted as m, plays a critical role in modulating the threshold wage by influencing node values. As depicted in Figure 4, the threshold wage exhibits a decline in response to an increase in the separation rate, but the relative wage hierarchy between nodes may vary with different values of m.

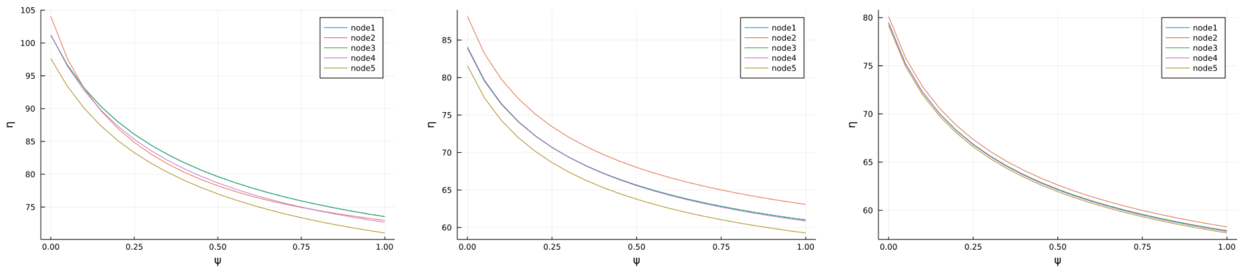
2.5.2 Career Path Illustration

Figure 5 outlines the career path of a typical worker within the network structure of example (a). Initially receiving unemployment compensation in period 2, the worker secures employment with employer 5. The wage makes her a settler in employer 5, but she is invol-

Network		Node 1	2	3	4	5	6
(a)	Ω	891.602	883.869	891.602	889.267	891.369	
	Λ	876.818	858.696	876.818	874.181	883.246	
	η	68.402	72.705	68.402	68.382	65.286	
(b)	Ω	880.473	873.275	880.473	878.322	880.158	808.285
	Λ	865.812	849.404	865.812	863.386	871.791	808.285
	η	67.577	71.368	67.577	67.557	64.626	55.772
(c)	Ω	896.592	890.306	896.157	895.196	896.218	
	Λ	888.511	864.629	881.774	887.141	888.401	
	η	65.627	73.384	68.530	65.518	65.478	
(d)	Ω	504.846	504.846	504.846	504.846	504.846	
	Λ	504.846	504.846	504.846	504.846	504.846	
	η	34.834	34.834	34.834	34.834	34.834	
(e)	Ω	556.897	556.897	556.897	556.897	556.897	556.897
	Λ	556.897	556.897	556.897	556.897	556.897	556.897
	η	38.426	38.426	38.426	38.426	38.426	38.426
(f)	Ω	903.497	903.497	903.497	903.497	903.497	
	Λ	899.556	899.556	899.556	899.556	899.556	
	η	64.176	64.176	64.176	64.176	64.176	
(g)	Ω	903.575	903.575	903.575	903.575	903.575	903.575
	Λ	901.721	901.721	901.721	901.721	901.721	901.721
	η	63.210	63.210	63.210	63.210	63.210	63.210

* Network (g), not in Figure 3, is a complete network with 6 nodes.

Table 1: Node values and threshold wage for each node within different network structure



(a) $m = 0.1$ (b) $m = 0.5$ (c) $m = 0.9$

Figure 4: Threshold wage η declines with higher separation rate ψ

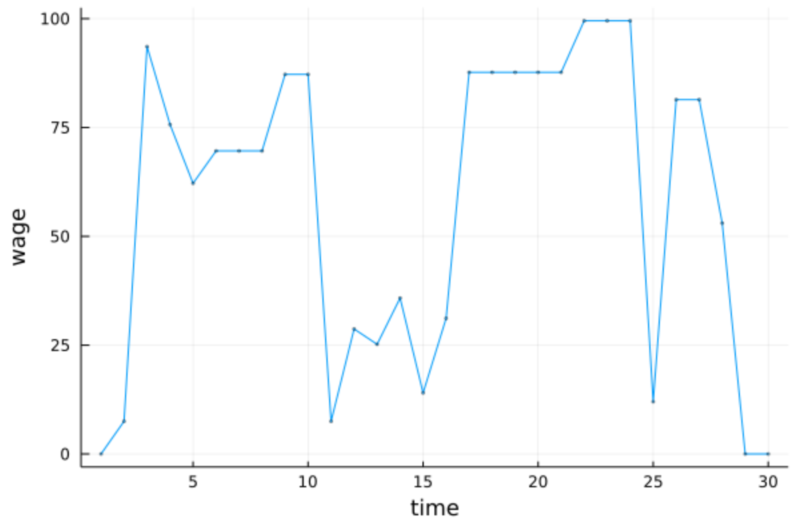


Figure 5: Wage dynamics of a typical worker in network (a). All parameters are the same.

untarily separated in period 3. Using the network connection of employer 5, she becomes a settler in employer 4 where she is involuntarily separated again in period 4. Then, she becomes a mover working for employer 2 in period 5. She quits her job 1 period later and moves back to employer 4. Note that, except the separation at period 11, all transitions are job-to-job from the data with no unemployment spell between them. The wage dynamics reflect her decisions to maximize the present value of lifetime income before exiting the labor market in period 29. Through this narrative, we observe how different positions within the employer network facilitate a sequence of job changes, each decision meticulously aligned with the goal of maximizing lifetime income.

2.5.3 Computation Problem

Computing node values in the above examples is facilitated by a fixed-point iteration algorithm. However, this approach encounters significant challenges as network size expands, leading to the 'curse of dimensionality'. When $\delta_L > 0$, the total possible combinations of the offers arrived can be represented by $\sum_{n=1}^{N-1} \binom{N-1}{n} = 2^{N-1} - 1$. For instance, in a network comprising 20 nodes, this formula yields $20 \times 2^{19} (> 10^7)$ combinations, necessitating the computation of more than ten million probability coefficients prior to iterations.

To alleviate computational demands, 3 additional assumptions can be introduced:

Network Scale Reduction: Concentrating on a more narrowly defined network can considerably reduce complexity. By examining networks at the industry level as opposed to the firm level, or prioritizing the firm level over the establishment level - given that a single firm may comprise multiple establishments - the number of nodes within the network is effectively decreased. However, this approach requires that the empirical relevance of the analysis is not unduly compromised.

Simplification of Arrival Rates: Adjusting arrival rates can further simplify calculations. Specifically, setting $\delta_L = 0$ eliminates the possibility of receiving offers from unlinked employers, thereby limiting the maximum number of offers a worker might receive to the number of their employer's connections. This reflects the practical reality that workers are unlikely to receive job offers from all employers in disparate geographical and industrial contexts.

Exploiting Symmetric Network Topologies: Exploiting symmetric topologies, such as empty or complete networks (as illustrated in panels (d)-(f) of Figure 3), or the peripheral nodes in a star network, can dramatically reduce computational dimensions. In such cases, all nodes share identical network positions and consequently identical node values, allowing for potential analytical solutions to be derived. This case also highlights the impact of high unemployment compensation: If it is set at a level that disincentivizes work, all employers become symmetrically irrelevant to workers, leading to a scenario where both the strong and weak node values for each employer converge to $\Omega_i = \Lambda_i = \frac{\gamma}{1-\beta(1-d)} (\forall i \in N_v)$.

3 Equilibrium and Network Structure Transition

In this section, we explore the model's equilibrium and demonstrate numerically how the economy transitions following a network structural change. Our simulations utilize the parameters of previous examples. We focus on the decision making process of workers, who are influenced by the node values and threshold wages delineated in Table 1.

3.1 Share of Settlers

Before deriving the steady states, we need to know the equilibrium share of settlers in each employer.

Lemma 1 *The share of settlers s_i in the employer i at steady state is*

$$s_i = \frac{1 - F_i(\eta_i)}{1 - xF_i(\eta_i)} \in (0,1), \tag{25}$$

where $x = (1-d)(1-\psi)$ is survival rate, and $F_i(z) = Pr(w < z|i)$ is the wage distribution of the unemployed workers who accept the offer from the employer i.

Proof. Define the number of workers in the employer i at the beginning of the period t as $W_t(i)$; the number of workers who enter the employer i in period t as $W_t^{in}(i)$. At steady state, $W_t(i) = W(i)$, $W_t^{in}(i) = W^{in}(i)$, so

$$W^{in}(i) = (1 - xs_i)W(i). \tag{26}$$

Meanwhile, the ratio of settlers and movers is also stable, i.e.

$$\frac{W(i)xs_i + W^{in}(i)Pr(w > \eta_i|i)}{W^{in}(i)Pr(w < \eta_i|i)} = \frac{s_i}{1-s_i}, \tag{27}$$

Together solve the share of settlers. ■

The share of settlers depends on the network position of employer i, as influenced by $F_i(z)$ and η_i. However, a direct comparison of the share of settlers between employers is not straightforward. For instance, employer i may not necessarily have a higher share of settlers than employer j, even if it occupies a more central position within the network. This is because while F_i may be first-order stochastically dominated by F_j, differences in threshold wages ($\eta_i < \eta_j$) can result in a varied share of settlers for more central nodes.[14]

3.2 Labor Flows and Employment in Steady State

In equilibrium, labor outflow and inflow from each node should be balanced. The labor outflow of one employer includes both voluntary and involuntary separations. In contrast, the labor inflow to an employer consists of inexperienced workers and movers from other employers. For an employer i, this equilibrium condition implies that the inflow of workers, $W^{in}(i)$,

[14] See Appendix B illustrating with the examples in the previous section.

matches the outflow, and can be expressed as:

$$W^{in}(i) = I \cdot P_{0i} + (1-d)\sum_{j\neq i}\left[(1-s_j)P_{ji}^{\Omega} + \psi s_j P_{ji}^{\Lambda}\right]W(j) \tag{28}$$

where I is the steady-state number of inexperienced workers at steady states, which is

$$I = \frac{b(1-\delta_L)^N L}{1-(1-d)(1-\delta_L)^N}, \tag{29}$$

where b stand for entry rate in labor force, L is the initial population size. [15] This suggests that I is independent of the internal connections of the network, highlighting its role in providing fresh labor force to all employers.

The variables P_{0i}, P_{ji}^{Ω}, and P_{ji}^{Λ} represent the transition probabilities for inexperienced workers moving to employer i, and for voluntarily and involuntarily unemployed workers from employer i accepting offers from employer i, respectively. These probabilities are determined through a series of equations integrating the distribution of wage offers and the decision-making process of workers based on their reservation values and competing offers. Formally,

$$P_{0i} = \delta_L\left\{\sum_{n=0}^{N-1}\sum_{k=1}^{\binom{N-1}{n}} p_1(k \mid n)\int\cdots\int_{D_1} f(w_{i_1},\cdots,w_{i_n},w_i)dw_{i_1}\cdots dw_i\right\}, \tag{30}$$

$$P_{ji}^{\Omega} = \delta_{ji}\left\{\sum_{n=0}^{N-1}\sum_{k=1}^{\binom{N-1}{n}} p_2(k \mid n)\int\cdots\int_{D_2} f(w_{i_1},\cdots,w_{i_n},w_i)dw_{i_1}\cdots dw_i\right\}, \tag{31}$$

$$P_{ji}^{\Lambda} = \delta'_{ji}\left\{\sum_{n=0}^{N-1}\sum_{k=1}^{\binom{N-1}{n}} p_3(k \mid n)\int\cdots\int_{D_3} f(w_{i_1},\cdots,w_{i_n},w_i)dw_{i_1}\cdots dw_i\right\}, \tag{32}$$

where $p_z(k|n)$ $(z=1,2,3)$ is the probability of the kth combination conditional on n offers arrived besides the offer from employer i, $f(w_{i_1},\cdots,w_{i_n},w_i) = f(w_{i_1})\cdots f(w_{i_n})f(w_i)$ is the joint probability density of these $n+1$ offers, and D_z $(z=1,2,3)$ is the domain where the

[15] Let I_t be the number of inexperienced workers, and L_t be the size of the population, at period t. For each period, inexperienced workers include newly born workers and inexperienced workers of the last period. The law of motion, therefore, is $I_{t+1} = (1-\delta_L)^N[b \cdot L_t + (1-d)I_t]$, where $(1-\delta_L)^N$ is the probability of no offer arriving. In steady state when $b = d$, $L_t = L$ and $I_{t+1} = I_t = I$. Therefore, the number of inexperienced workers in the steady state depends on the death rate, the birth rate, the population size, the lowest arrival rate, and the number of employers in the network, but not on the edges inside the networks. In the case of $b > d$ or $b < d$, the number of inexperienced workers fluctuates similarly to the general population.

unemployed worker to move to the employer i, i.e.

$$D_1 = \Big\{ (w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_0 < w_i + \beta(1-d)V(i, w_i) \quad \&$$
$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n})\} < w_i + \beta(1-d)V(i, w_i) \Big\}$$
$$D_2 = \Big\{ (w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_j^{\Omega} < w_i + \beta(1-d)V(i, w_i) \quad \&$$
$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n},)\} < w_i + \beta(1-d)V(i, w_i) \Big\}$$
$$D_3 = \Big\{ (w_{i_1}, \cdots, w_{i_n}, w_i) | \bar{V}_j^{\Lambda} < w_i + \beta(1-d)V(i, w_i) \quad \&$$
$$\max\{w_{i_1} + \beta(1-d)V(i_1, w_{i_1}), \cdots, w_{i_n} + \beta(1-d)V(i_n, w_{i_n})\} < w_i + \beta(1-d)V(i, w_i) \Big\}$$

where i_n indicates one of the n arrived offers dominated by the offer from employer i. The value of employer i's offer must be larger than both the reservation value and the values of other competing offers.

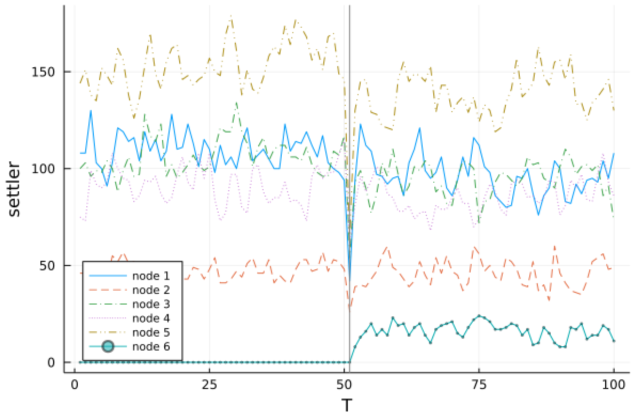
The equilibrium employment level for each employer, $\hat{W} = [W^*(1), \ldots, W^*(N)]'$, can be obtained by plugging equation 30-32 into the equation 28, together with the equation 26. The equilibrium is achieved when $\hat{W}$ satisfies the fixed point in the system of equations:

$$[1 - xs_1, 1 - xs_2, \cdots, 1 - xs_N]\hat{W}^* = \begin{bmatrix} (1-xs_1)W^*(1) \\ (1-xs_2)W^*(2) \\ \vdots \\ (1-xs_N)W^*(N) \end{bmatrix} = H(\hat{W}^*) \tag{33}$$

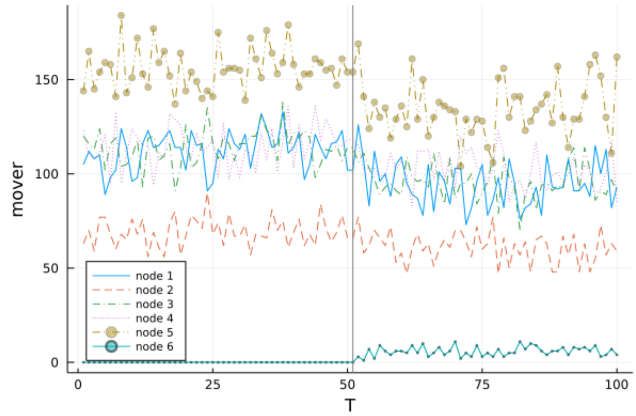
where function H satisfies equation 28 for each $i \in N_v$. It is easy to show the existence and uniqueness of the solution to this system of linear functions. This model underscores how central nodes within a network are poised to attract more employees in the long run, as illustrated in panel (b) of Figure 9 and further supported by Table 2 in Appendix A.

3.3 Structural Change of Network

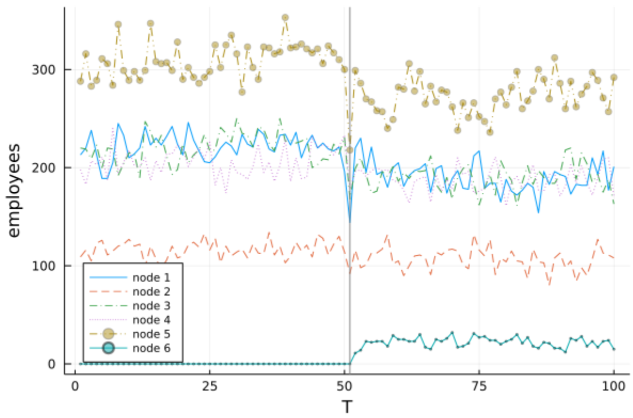
This section explores the dynamics of employment and the share of settlers in response to structural change in the network. We conduct simulations under two distinct scenarios to understand the changes in network configurations and their implications on employment metrics. The initial economy includes 2000 inexperienced workers and unfolds over 100 periods. At time $t = 51$, a shock changes the structure of the network.



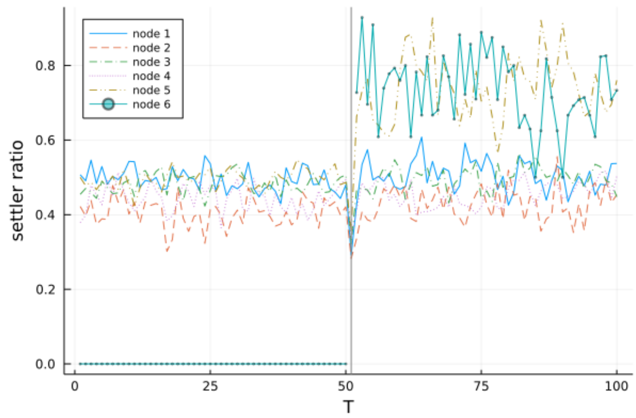
(a)



(b)

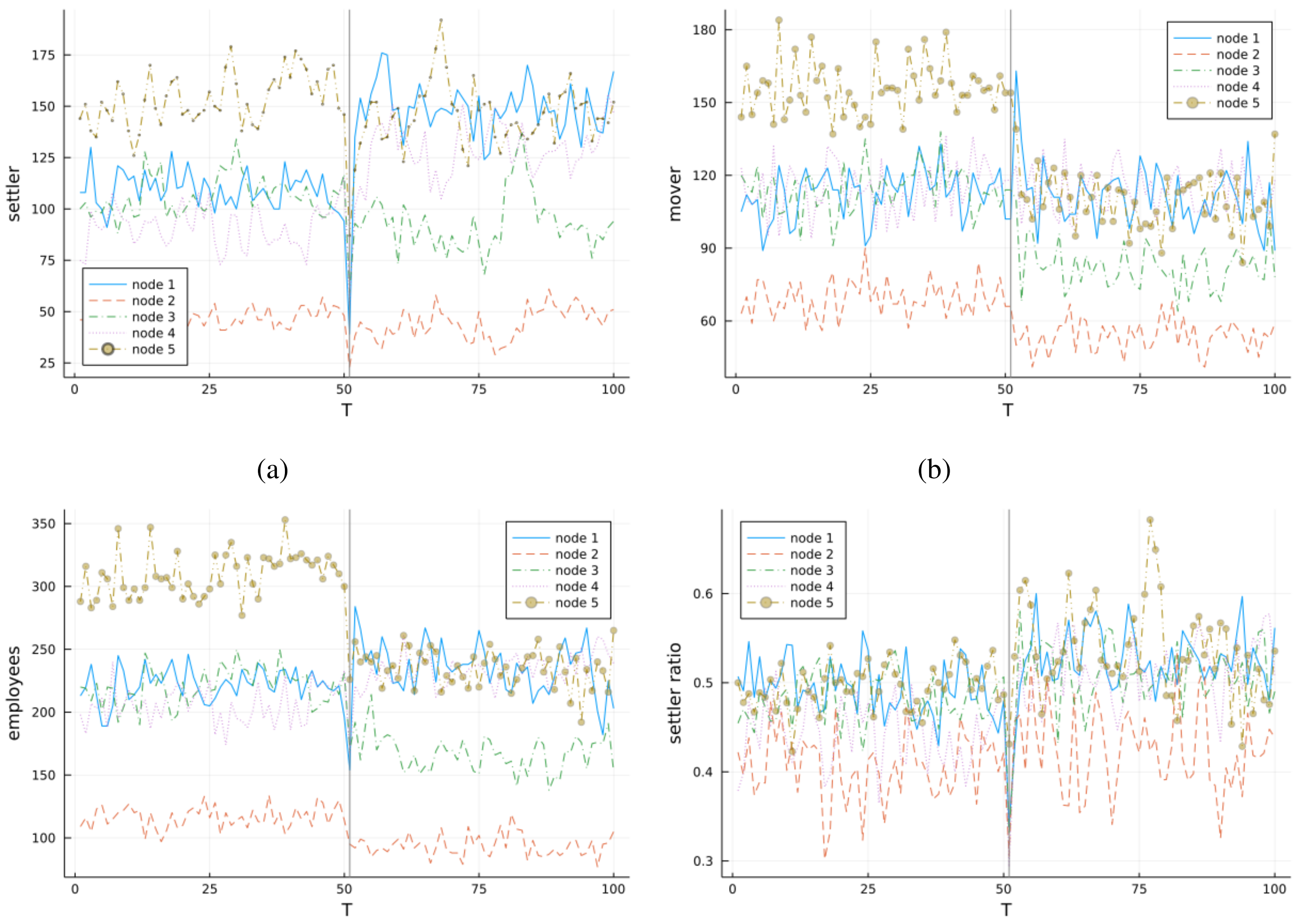


(c)



(d)

Figure 6: Transitions from network (a) to (b)



(a) (b)

(c) (d)

Figure 7: Transitions from network (a) to (c)

In the first case, the shock introduces a new employer in the network. This scenario is depicted through the transition from network configuration (a) to (b), as illustrated in Figure 3, with the specific transition shown in Figure 6. The shock alters the node values across the network and prompts workers to reassess their career decisions. This reassessment results in noticeable fluctuations in the ratios of settlers to movers, along with changes in the total number of settlers, movers, and employed individuals. The emergence of a new, albeit less appealing, employer still attracts a flow of workers. Over time, the initial adverse impacts of the shock on employers' workforce sizes diminish. However, certain nodes, specifically nodes 1 and 5, fail to recover to their pre-shock employment levels.

The second scenario investigates the effects of adding a new edge to the network, shown from example (a) to (c) as shown in Figure 3, with the transition detailed in Figure 7. The network structural shock results in an increasing number of settlers in nodes 1 and 4. This is because (i) a new edge is constructed between them and significantly makes them more attractive to workers; (ii) the threshold wages of both employers decrease after the shock, and thus result in a higher (lower) settler(mover) ratio in steady state. Moreover, the introduction of the new edge disrupts the prevailing network hierarchy, particularly diminishing the advantage of node 5's network position by enlarging the node degree of nodes 1 and 4.

4 Discussion of The Network Effect

The structure of the employer network - including the number of nodes (employers), the edges connecting them, and the overall topology - plays a pivotal role in shaping economic outcomes in the labor market. Central to this analysis is the premise that the node values navigate workers' decisions. Thus, this section discusses the influence of employer network structures on node values.

4.1 Network Effect on Node Values

We start with the effect of additional edges in the network on node values.

Lemma 2: *The node values in a network weakly increase with the addition of edges.*

Proof. The lemma is substantiated through two primary arguments: (1) under certain conditions, the addition of an edge does not alter node values; (2) barring these conditions, the introduction of a new edge results in an increase in node values.

Node values may remain unchanged in the following 3 cases. First, when $\delta_L = 0$, an employee from an isolated employer does not receive alternative job offers upon resignation, leaving the isolated employer's node value unaffected by network changes. Second, an exceedingly high unemployment compensation diminishes work incentives. In scenarios where all workers opt for unemployment, the employer network's structure becomes irrelevant. Third, as $(\delta_H - \delta_L) \to 0$, the effect of an additional edge vanishes.

If all other conditions are well defined, the new edge improves the node values. Consider a new edge connecting node i and node j. This edge, by definition, increases the node value of employer i, employer j, or both, depending on whether the network is directed or undirected. Following Proposition 3, this increment extends to adjacent nodes and their neighbors, propagating throughout the network. ■

The intuition is that an enriched network offers workers better future prospects, increasing their expected values. Consequently, Lemma 2 suggests a universal benefit for all employers in the network, including those not directly connected. When $\delta_L > 0$, even the node value of an isolated employer would increase. The comparative analysis of the network configurations in Figure 3 and Table 1 illustrates that an additional edge (e.g., between nodes 1 and 4) universally increases the node values. Hence, within a given node number, a complete network boasts the highest node values, whereas an empty network has the lowest.

The examples in Figure 3 illustrate this property. Starting with an empty five-node network (graph d), which exhibits the lowest node values, the introduction of edges or nodes incrementally increases these values, as demonstrated in networks (a) and (e). Network (b) emerges from expanding network (a) with an additional node or augmenting network (e) with new edges, situating its node values above those of network (e) but below those of network (a). The progression from example (a) to (c) culminates in the highest node values within the completely interconnected network (f).

However, the effect of increasing the number of nodes on the node values is ambiguous and depends on the particular structure of the network. Although adding a node to an empty network (as shown in graphs (d) and (e)) universally boosts node values, the impact varies in networks within irregular structure, such as in examples (a) and (b). Here, a new node may dilute the value of existing nodes by rendering the network sparser.

Finally, the network structure per se could affect the node values even with the same

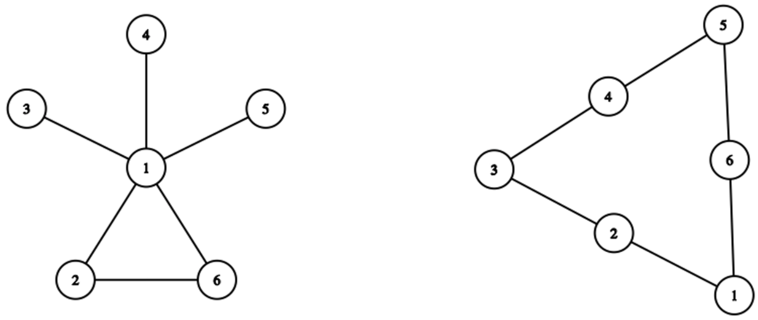


Figure 8: Two network structures with 6 nodes and 6 edges

number of nodes and edges. To get a sense of the nuanced effect of network topology, consider two networks with 6 nodes and 6 edges, but distinct topological structures in Figure 8. There is no difference in node values in the regular network on the right, while employer 1 in the left irregular network enjoys the highest node value. As node values govern working decisions and overall wage distribution, this complexity highlights another source of wage inequality - expected opportunities provided by employers and their network positions, necessitating a detailed examination of network structure and their economic implications.[16]

5 Conclusion

This paper constructs a simple search model with employer network that brings heterogeneous working prospects for workers contingent on employer's particular position in network. The model reveals that the network structure inherently shapes dynamics in labor market.

Our model emphasizes that the employer network position could be important to workers in providing long-term prospects. Central employers are the ones with more connections or neighbors that are also central. Our model suggests that a worker may move to a new employer with a wage cut because the employer could provide more opportunities of moving to another employer with a higher wage.

While the model assumes that employer networks structure os exogenously given, the genesis of these networks remains an open question. The literature suggests that social networks

[16]The notion of network symmetry is captured by the mathematical concept of graph automorphism. An automorphism of the graph σ is then a permutation, or relabeling, of the vertices $v \mapsto \sigma(v)$ such that $(\sigma(i), \sigma(j)) \in E$ is an edge only if $(i, j) \in E$ for all i, j. More symmetric nodes result in a higher redundancy ratio of the network.

among coworkers and the production networks of employers could both play significant roles in expanding workers' opportunities for upward mobility. This confluence of social and production networks likely shapes the labor market landscape and offers a rich avenue for future research.

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Network		Node 1	2	3	4	5	6
(a)	Settlers	109.4	47.24	106.46	91.46	153.44	
	Movers	112.4	69.3	114.24	114.74	155.88	
	Employees	221.8	116.54	220.7	206.2	309.32	
	share of settlers	0.494	0.406	0.482	0.443	0.496	
(b)	Settlers	102.88	45.8	99.98	89.86	142.22	15.68
	Movers	101.22	62.92	100.54	102.88	136.4	5.36
	Employees	204.1	108.72	200.52	192.74	278.62	21.04
	share of settlers	0.504	0.422	0.499	0.466	0.510	0.744
(c)	Settlers	158	49.94	100.5	138.62	153.52	
	Movers	129.7	59.72	99.18	132.52	128.84	
	Employees	287.7	109.66	199.68	271.14	282.36	
	share of settlers	0.552	0.442	0.502	0.507	0.549	

Table 2: 50-Period Averages

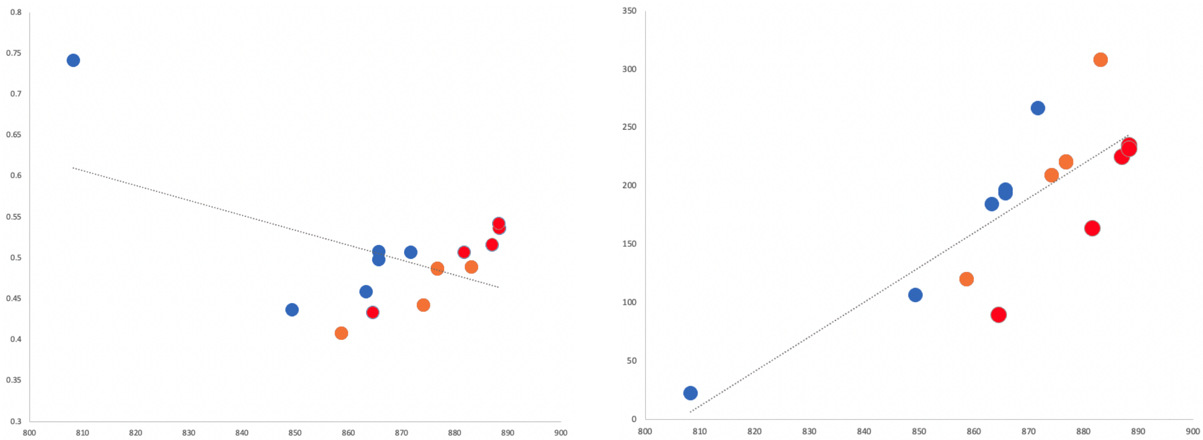
Appendices

A 50-Period Averages

Table 2 presents the simulated average number of settlers, movers, employees, and share of settlers, for each node from time 50 to 99.

B Relationship between Node Values and share of settlerss

Figure 9 panel (a) illustrates the relationship between node values and share of settlerss across employers in networks (a), (b), and (c). In panel (a) and (b), x-axes are (weak) node values, while y-axes represent average share of settlers and average number of employees, respectively. The average numbers cover the last 50 periods. Dots in orange, blue, and red, are for example (a), (b), and (c), respectively. In panel (a), the regressing line is $Y = 2.0845 - 0.001824X$, with $p = 0.05911$. After excluding the new node from the network (b), the regression line



(a) share of settlers and node values

(b) Number of employees and node values

Figure 9

becomes $Y = -1.7771 + 0.002589X$, with $p = .00086$; In panel (b), the regressing line is $Y = -2396.2234 + 2.9725X$, and $Y = -2870.5042 + 3.5145X$ after excluding the isolated node 6, with $p = 0.00012$ and $p = .004$, respectively. The results have little difference with the strong node value (Ω). Notably, central nodes exhibit higher share of settlerss upon excluding the new node from example (b), though the justification for such exclusion remains debatable.